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# The size of films in dry foams

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**Abstract**

The relationship between the size of films in a dry foam and the size of the bubbles is investigated. The study was carried out using foam structures simulated using Surface Evolver, with the structures covering a wide range of poly-dispersities. It was found that the most important factor influencing the size of a film is the size of the smaller bubble to which it is attached. The larger bubble does have an influence, but it is much smaller, with the film size increasing by approximately 80% as the larger bubble goes from the same size as the smaller bubble to infinitely large. The relationship between a film's size and the size of the two bubbles to which it is attached was found to be independent of the underlying bubble size distribution, with the probability distribution for the size of the film depending only on the size of the neighbouring bubbles.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

The topological and geometrical structure of foams has received a lot of attention in the literature [1–5] as the structure of foam is critical to many of its properties, including its rheology, drainage, coarsening and many others. Most previous work has concentrated either on the purely topological properties of the foam, such as the number of faces per bubble or edges per facet [1, 2] or has looked at the average geometric properties, such as the specific surface area of the foam or the length of Plateau borders per volume [3]. One aspect of foam geometry that has not received much attention is the size distribution of the individual films within the foam. In particular, the coalescence of bubbles in the foam depends quite strongly on this size distribution. This is because the life time of films, and thus the film failure frequency, depends very strongly on the size of the films. For example, as predicted by the Reynolds equation, the thinning rate of the film is inversely proportional to the squared radius of the film for small films with immobile interfaces (e.g. [6, 7]), while other types of films (e.g. larger films or films with more mobile interfaces) have dependencies on film size that range between this squared relationship and a linear relationship.

In this paper, the coupling between the size distribution of the bubbles and relative sizes of the films will be investigated. While it is possible to investigate the internal structure of foams experimentally by using, for instance, direct optical observation [8, 9] or optical tomography [10], it has been shown that for dry foams computer simulations can very accurately predict the structure [11–15]. Due to the ease with which geometric information can be subsequently extracted from these simulations, a method similar to that proposed by Kraynik [12–14] will be employed in this work. The main aim of this paper is to investigate how the sizes of the films are influenced by the sizes of the bubbles to which they are attached.

## 2. Methodology and simulations

By assuming that the foam is in the dry limit, the thin liquid film degenerates to 2D interfaces constrained by Plateau's laws. The bubbles are thus trivalent polyhedra. The initial structure of the foam is produced by applying a Voronoi tessellation onto a 3D cubic domain. In order to produce a foam structure with a given level of poly-dispersity, the volume of every bubble in the foam is gradually adjusted according to a skewed normal distribution. The obtained foam, with assigned target poly-dispersity, is then relaxed and annealed in a manner similar to that employed by Kraynik [11].

Foams were produced that were either fully periodic or periodic in two directions, with two opposite free surfaces in the third direction. The method for producing foams with

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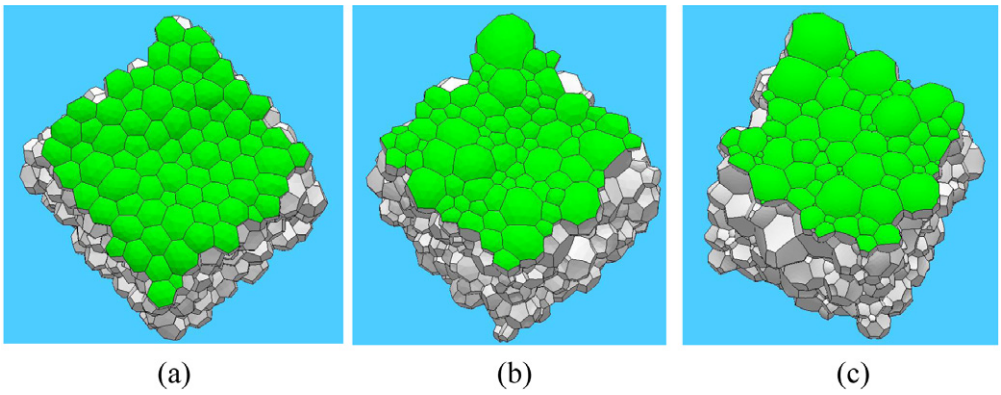



**Figure 1.** Structure of the foams with different poly-dispersities ((a) NSD = 0.1; (b) NSD = 0.4; (c) NSD = 0.7).

free surfaces has been described in a previous paper [15]. This paper also showed that the structure of the surface bubbles produced by this method is very similar to that found experimentally by Matzke [8]. Furthermore, in both this work and in previous work [15] it was found that the internal bubbles of the foam with the free surface produce the same results as the fully periodic foam and they are thus not reported separately.

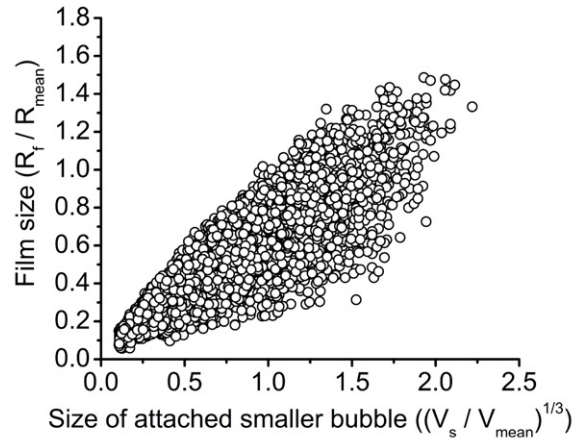
In all the simulations in this work a similar number of bubbles are used (590 ± 10). This number was chosen so that at even the highest levels of poly-dispersity used, the results are very similar to those achieved with twice the number of bubbles. Since computational cost increases dramatically with increasing number of bubbles, large sample sizes are achieved by repeating simulations with the same level of poly-dispersity and combining the data rather than trying to produce enough data in a single simulation.

The poly-dispersity of bubbles is represented by the normalized standard deviation (NSD) of the equivalent spherical radius of the bubbles. Seven different levels of poly-dispersity are studied in this work, ranging from mono-dispersed to a NSD of 0.7. In the highest poly-dispersity simulations, the largest bubbles are approximately 10 000 times the volume of the smallest. Figures 1(a)–(c) show the structure of three representative foams with poly-dispersities of NSD = 0.1, 0.4 and 0.7.

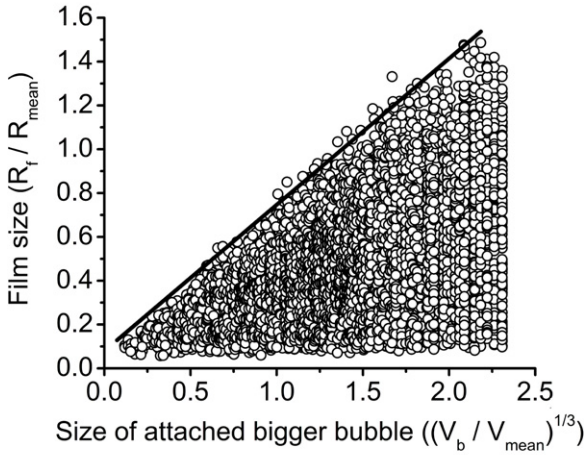
## 3. Results and discussion

### *3.1. The effect of the size of the neighbouring bubbles on the film size*

Figures 2(a) and (b) show the variations of the film size with the size of bubbles attached to the film, when the underlying poly-dispersity of the bubbles is 0.7. The film size is represented by the equivalent circular radius of the film $R_f$, which is the square root of the area divided by $\pi$ of the curved film separating two bubbles. It is normalized by the equivalent spherical radius $R_{mean}$ of a bubble with the mean volume of all bubbles in the simulation. The volumes of the bigger and smaller bubbles attached to the film are denoted by $V_b$ and $V_s$, which are normalized by the mean volume of bubbles in the overall foam




(a)




(b)

**Figure 2.** The variations of the film size with the size of either the smaller bubble (a) or the bigger bubble (b) attached to the film.

$V_{mean}$. There are a number of interesting points to note from these figures: firstly, there is not a deterministic correlation between the size of the film and the size of the bubbles attached to the film. There is a range of possible film size for a given size of either the smaller bubble or the larger bubble. Secondly, the film size is strongly dependent on the size of the smaller

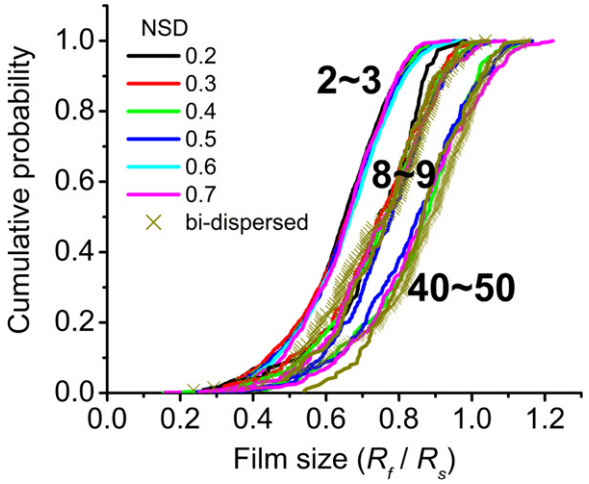



**Figure 3.** Cumulative probability distribution of the internal film size for three different volume ratio bin classes at different poly-dispersities.

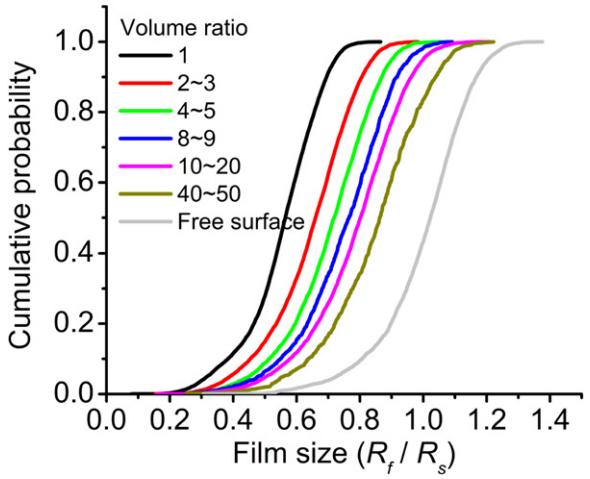



**Figure 4.** Cumulative probability distribution of the internal film size for different volume ratio bin classes.

bubble attached to the film, while having no easily discernible relationship to the size of the bigger bubble. Figure 2(a) also shows that while there is scatter in the relationship, the effect of the size of the smaller bubble on the film size is linear. Figure 2(b) shows that for any given size of the bigger bubble, the film size can adopt almost any value below a certain threshold. The threshold is simply due to the fact that the film cannot be much larger than the size of the larger bubble to which it is attached, but it can be much smaller.

Since the film size is found to be strongly dependent on the size of the smaller bubble attached to the film and the dependence is roughly linear, in the subsequent analysis the size of the film is normalized by the size of the smaller bubble to which it is attached.

### *3.2. The effect of the larger bubble on the film size*

While the previous section has demonstrated that the biggest influence on a film's size is the size of the smaller bubble to which it is attached, this section will look at the influence of the size of the larger bubble. In order to carry out this investigation the films were divided into separate categories according to the volume ratio of their neighbouring bubbles. Figure 3 shows the cumulative probability distribution obtained for the size of the films for three different volume ratio bin classes at different underlying poly-dispersities of the bubbles in the foam. In figure 3, the film size $R_f$ is normalized by the equivalent spherical radius of the attached smaller bubble $R_s$ ($V_s = \frac{4}{3}\pi R_s^3$). While there is a small amount of scatter between the curves for different levels of underlying bubble poly-dispersities, especially for the larger ratios where the sample size becomes smaller, there is no systematic effect of the underlying poly-dispersity on these relationships. To test this, an extreme case was investigated where a single larger bubble was imbedded in an otherwise mono-disperse foam. In order to obtain sufficient data, the results from a number of these simulations were combined (shown as 'bi-dispersed' on figure 3). The cumulative distributions from these bi-dispersed foams follow exactly the same relationship as that from the various poly-dispersed foams. This further reinforces the fact that, for a given ratio of the volume of the larger to smaller attached bubbles, the shape of this distribution is independent of the underlying poly-dispersity.

Since these relationships are independent of underlying poly-dispersity, the data from all the different simulations are combined, with the results from a wide range of different volume ratio bin classes plotted in figure 4. Included with these data are the results for the films at the free surface of those simulations that were not periodic in one of the directions. These free surface films are equivalent to an infinite volume ratio between the larger and smaller bubble. In a previous paper that concentrated solely on these free surface films [15], it was found that there is a probability distribution for the ratio of surface film size to attached bubble size that is independent of the underlying poly-dispersity, which is consistent with this work.

In figure 4 it can be seen that there is steady progression in the probability distribution from that in mono-dispersed foams to that at the free surface. There are subtle changes in the shape of this distribution as the volume ratio of the neighbouring bubbles changes, but the most noticeable aspect of the behaviour is that the mean film size increases. Figure 5 shows the mean film size as a function of the volume ratio of neighbouring bubbles to which the film is attached. Firstly, the mean film size is found to increase as the volume ratio of the neighbouring bubbles increases. This effect is relatively small, though, with the average film size increasing by roughly 80% as the volume ratio of bubbles increases from 1 to infinity. Secondly, the error bars, which represent one standard deviation of the film size normalized by the smaller bubble size, do not systematically change with different volume ratio of the neighbouring bubbles. This is consistent with the main change in the cumulative probability function (figure 4) being a shift in the mean, with very little change in the shape. This implies that to collapse these curves to a single line, they should be shifted according to the mean, rather than scaled. To do this an expression for the mean film size needs

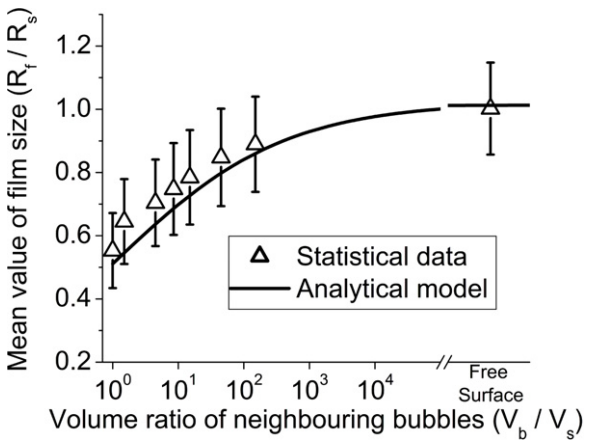



**Figure 5.** Mean normalized film size as a function of volume ratio of neighbouring bubbles based on either the structure simulation or the analytical model (equation (4)).

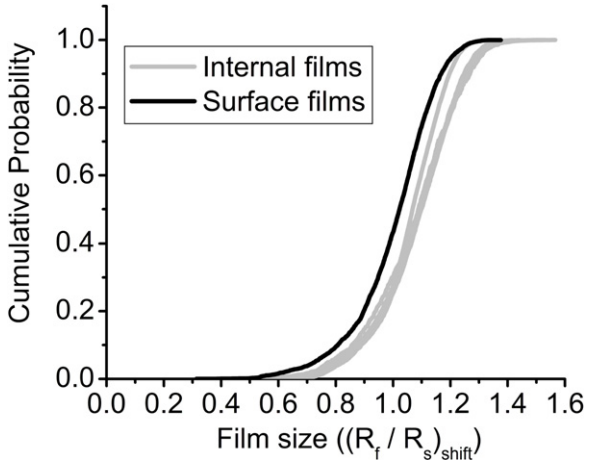



**Figure 6.** Cumulative probability distribution of the shifted internal film size and that of the original surface film size.

to be developed. This will be done using simple geometric arguments.

### *3.3. Geometric model for the mean film size*

The relationship between the mean $R_f/R_s$ and the ratio of the size of the neighbouring bubbles can be explained using a simple model. For this model it will be assumed that there is a large bubble, radius $R_b$, surrounded by a layer of smaller bubbles of radius $R_s$. If it is assumed that the centres of the smaller bubbles fall on a spherical shell of radius $(R_s + R_b)$, then the number of bubbles on the shell, $N$, is the following (noting that in a dry foam the bubble shell covers the entire area):

$$N = \frac{4\pi(R_s + R_b)^2}{a\pi R_s^2}, \tag{1}$$

where the constant $a$ is O(1) and is the factor by which the cross-sectional area of the smaller bubble is larger than that of a circle of equivalent radius. The average film area, $A_f$, is the surface area of the larger bubble divided by the number of smaller bubbles on the surface:

$$A_f = \frac{b4\pi R_b^2}{N}, \tag{2}$$

where the constant $b$ is also O(1) and is the factor by which the surface area of the large bubble is larger than that of an equivalent sphere. If the size of the film is represented by its equivalent circular area (i.e. $A_f = \pi R_f^2$), then:

$$\frac{R_f}{R_s} = K\frac{\frac{R_b}{R_s}}{\left(\frac{R_b}{R_s} + 1\right)}, \tag{3}$$

where $K = \sqrt{ab}$ and is therefore also O(1). This can also be expressed in terms of the ratio of the bubble volumes:

$$\frac{R_f}{R_s} = K\frac{\left(\frac{V_b}{V_s}\right)^{1/3}}{\left(\left(\frac{V_b}{V_s}\right)^{1/3} + 1\right)}. \tag{4}$$

$K$ also represents the average ratio of the film size to bubble size at the free surface of the foam (where $V_b/V_s = \infty$). At the free surface it has been found that the mean value of $R_f/R_s$ is 1.002 and thus $K$ takes this value accordingly. The rest of the relationship is thus predicted with no further free parameters. In figure 5, equation (4) is represented by the continuous curve and is found to closely approximate the data produced by the structural simulations using Surface Evolver.

### *3.4. Relationship between the size distribution of the internal films and that of the surface films*

Figures 4 and 5 show that the mean value of the ratio of the film size to the smaller bubble size increases as the volume ratio of the neighbouring bubbles increase, while the shape and width of its probability distribution remains almost unchanged. This implies that this distribution for the internal films can be collapsed onto that for the surface films, by shifting the curves. The magnitude of the shift is the difference between the mean size of internal films for a given combination of neighbouring bubble sizes (finite $V_b$ and $V_s$) and the mean size of the surface films, at which $V_b/V_s$ is infinite. The magnitude of the shift required can be calculated using equation (4):

$$M = \frac{K}{\left(\left(\frac{V_b}{V_s}\right)^{1/3} + 1\right)}. \tag{5}$$

The data from figure 4 can thus be collapsed by using the following transformation for the $x$ axis:

$$\left(\frac{R_f}{R_s}\right)_{\text{shift}} = \frac{R_f}{R_s} + M. \tag{6}$$

Figure 6 shows that this transformation collapses the data quite well and thus from this relationship, the distribution of sizes that a film can adopt can be calculated for any combination of neighbouring bubble sizes.

## 4. Conclusions

From this work we can conclude that it is the local bubble size rather than the global poly-dispersity of bubbles that is the dominant factor in determining the film size in the foam. The biggest factor influencing the size of a film in a dry foam is the size of the smaller bubble to which it is attached. The size of the larger bubble is shown to have an influence, but that influence is much weaker. The distribution of sizes that a film can adopt for a given combination of neighbouring bubble sizes is independent of the underlying bubble size distribution. This distribution for any combination of neighbouring bubble sizes can be collapsed onto a single curve.

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